Structural-Morphism System Property: $_{atis}$Homomorphismness

(Structural-morphism system properties are those properties that are part of the theory and define the mapping-relatedness of object-set components.)

Homomorphismness (general morphismness), $\mathscr{M}$, $=_{df}$ components that have the same connections as other components.

$$\mathscr{M} =_{df} \mathscr{M}(\mathbb{S}_1, \mathbb{S}_2) \mid [\mathbb{S}_1(P(\mathcal{A}_1)) \equiv \mathbb{S}_2(P(\mathcal{A}_2))]$$

Homomorphismness is a morphism; such that, the mapping is defined by equivalent affect-relation set predicates of each system.

Every affect relation of a system defines a **homomorphism** that is self-mapped. Between two systems, a **homomorphism** is defined by affect-relation sets that are defined by the same predicate.

HOMOMORPHISM or GENERAL MORPHISM: Functions between *structures* that preserve relations are called *homomorphisms*.

STRUCTURE: Let τ be a signature.
A *signature* is the collection of a set of constant symbols, and a set of n-ary relation symbols and a set of n-ary function symbols.
For $\mathcal{ATIS}$, the *signature* is: $\mathcal{V}$ = { a, b, c, …}; $\mathcal{R}$ = {{{x},{x,y}}, =, ⊂, ≡, ∀, ∃, ι, ŵ, A}, where {{x},{x,y}}∈$\mathcal{A}$; and $\mathcal{F}$ = {∈, ∪, ∩, \}; and additional relations or functions as required.

A *τ-structure*, $\mathfrak{a}$, consists of an object-set, $\mathfrak{U}$, called the *universe* (or *domain*) of $\mathfrak{a}$, together with

- For each constant symbol c∈τ, a component, $c^{\mathfrak{a}} \in \mathfrak{U}$;
- For each n-ary relation symbol R∈τ, a subset, $R^{\mathfrak{U}} \subseteq \mathfrak{U}^n$; and
- For each n-ary function symbol f∈τ, a function, $f^{\mathfrak{U}}: \mathfrak{U}^n \to \mathfrak{U}$.

When the context makes it clear in which structure we are working, we use the elements of τ to stand for the corresponding constant, relation, or function. When τ is understood, we call $\mathfrak{a}$ a *structure*, instead of a τ-structure. Also, it is common to write a∈$\mathfrak{a}$ instead of a∈$\mathfrak{U}$.
For $\mathcal{ATIS}$, the system relations are defined by the family of affect relations, $\mathcal{A}$.

Therefore, a *homomorphism* for *ATIS* is a function, or mapping, between two object-sets that, by definition, have the above properties of a *structure*. Now let's consider two object-sets in the same or different systems with respect to the same affect relation. Let Σ be a fixed signature, and $\mathscr{U}$ and $\mathscr{B}$ are two structures, object-sets, for Σ. A *fixed signature* simply means that both systems of *ATIS* have the same constants, relations and functions. And, while there may be numerous functions between these affect relations, the interesting functions between them; that is, from $\mathscr{U}$ to $\mathscr{B}$ are the ones that preserve the structure; that is, preserve the affect relations with respect to each object-set. Essentially, this means that we are concerned with comparable affect relations in both systems with respect to the respective object-sets. Now, homomorphism can be more carefully defined.

A function $f{:}\mathscr{U} \to \mathscr{B}$ is said to be a *homomorphism* if and only if:

1. For every constant symbol c of Σ, $f(c^{\mathscr{U}}) = c^{\mathscr{B}}$. That is, every constant in $\mathscr{U}$ is mapped to a constant in $\mathscr{B}$.
2. For every n-ary function symbol $\mathcal{F}$ of Σ,

$$f(\mathcal{F}^{\mathscr{U}}(a_1,a_2,\ldots,a_n)) = \mathcal{F}^{\mathscr{B}}(f(a_1),f(a_2),\ldots,f(a_n)).$$

   That is, the function; for example, the union, ∪, of the components in $\mathscr{U}$ is equal to the union of the functions in $\mathscr{B}$.

3. For every n-ary relation symbol $\mathcal{R}$ of Σ,

$$\mathcal{R}^{\mathscr{U}}(a_1,a_2,\ldots,a_n) \supset \mathcal{R}^{\mathscr{B}}(f(a_1),f(a_2),\ldots,f(a_n)).$$

   That is, the relation; for example, the subset relation, ⊂, of the components in $\mathscr{U}$ imply that the corresponding functions in $\mathscr{B}$ are subsets of $\mathscr{B}$.

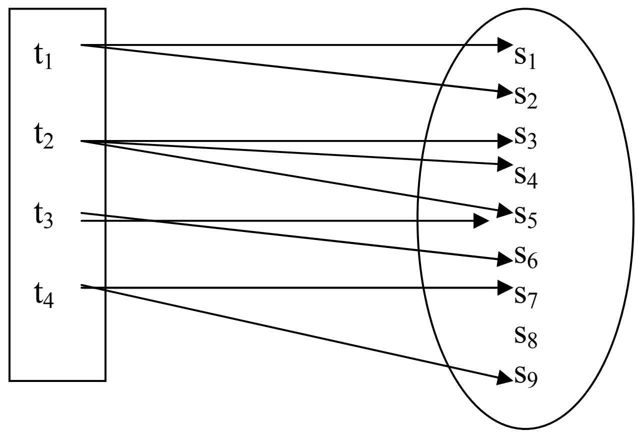
Therefore, a function, $f{:}\mathscr{U} \to \mathscr{B}$ is said to be a *homomorphism* if and only if relations with respect to each system are preserved under the mapping.

For example, consider two object-sets consisting of teachers, $\mathscr{T}$, and students, $\mathscr{S}$. Let $f{:}\mathscr{T} \to \mathscr{S}$ be a function between these two object-sets. We say that f is an *ATIS System Homomorphism* if:

- Every component of $\mathscr{T}$ is mapped to a component of $\mathscr{S}$. $f(c^{\mathscr{T}}) = c^{\mathscr{S}}$;
- Every function in $\mathscr{T}$ is preserved in $\mathscr{S}$. $f(\mathcal{F}^{\mathscr{T}}(a_1) = \mathcal{F}^{\mathscr{S}}(f(a_1))$; and
- Every relation in $\mathscr{T}$ is preserved in $\mathscr{S}$. $\mathcal{R}^{\mathscr{T}}(a_1) \supset \mathcal{R}^{\mathscr{S}}(f(a_1))$.

The following mapping represents an *ATIS System Homomorphism* designated by the affect relation ‘*guides the learning of*’. All of the above properties are preserved under this mapping. By the definition of an *ATIS Structure*, any mapping defined by an affect relation is an *ATIS System Homomorphism*.

Object-Set $\mathcal{T}$ **Object-Set $\mathcal{S}$**



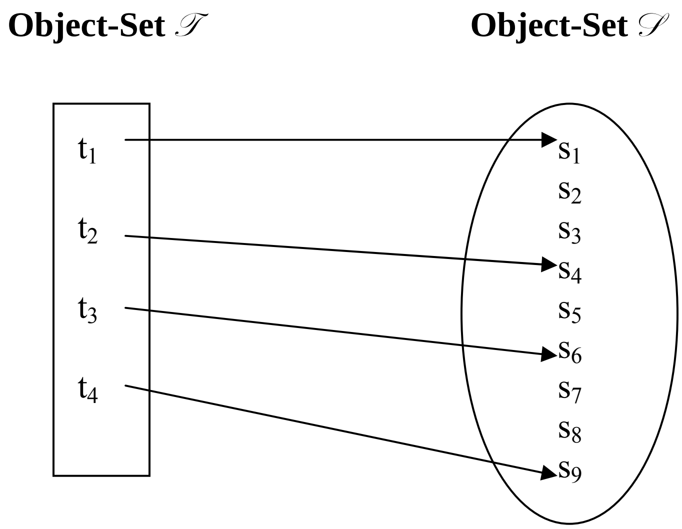
In *ATIS*, relations are defined by affect relations the components of which are represented by a set of two sets as follows: $\{\{x\}, \{x,y\}\}$. This representation designates that the affect relation is defined from ‘x’ to ‘y’. Therefore, if the above homomorphism represents the affect relation, $\mathcal{G}$, “guides the learning of,” $\mathcal{A}_{\mathcal{G}}$, for systems $\mathcal{T}$ and $\mathcal{S}$, then the affect relation, the homomorphism, is represented as:

$$\mathcal{A}_{\mathcal{G}} = \{\{\{t_1\},\{t_1,s_1\}\}, \{\{t_1\},\{t_1,s_2\}\}, \{\{t_2\},\{t_2,s_3\}\}, \{\{t_2\},\{t_2,s_4\}\}, \{\{t_2\},\{t_2,s_5\}\}, \{\{t_3\},\{t_3,s_5\}\}, \{\{t_3\},\{t_3,s_6\}\}, \{\{t_4\},\{t_4,s_7\}\}, \{\{t_4\},\{t_4,s_9\}\}\}$$

As a result of the definitions of an *ATIS Structure*, *ATIS System Homomorphism* with functions that have interesting properties can be identified by those properties just as would be in any other mathematical structure. As a result, the following morphisms are defined.

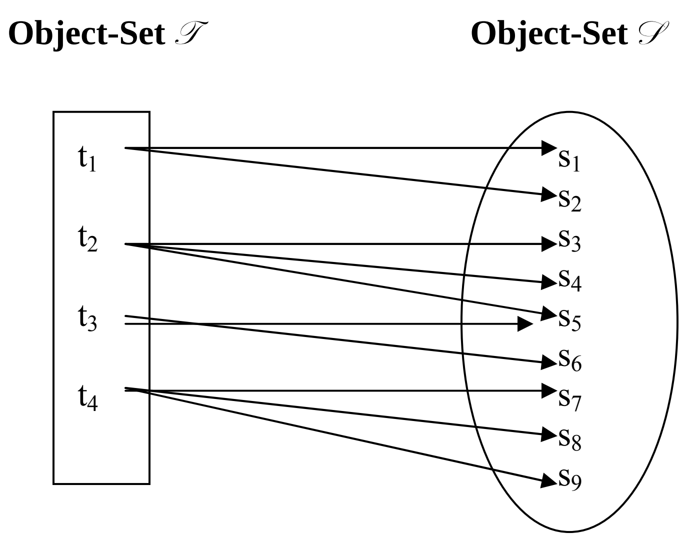
Monomorphism: A homomorphism that is an *injective function*; that is, a function that is *one-to-one*, is a *monomorphism*.

The following homomorphism, $f_{\text{mono}}:\mathscr{T}\to\mathscr{S}$, defines a *monomorphism*:



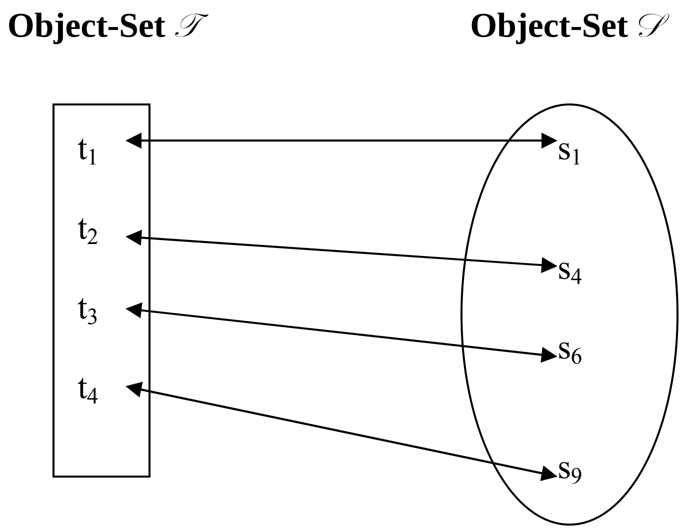
Epimorphism: A homomorphism that is a *surjective function*; that is, a function that is *onto*, is an *epimorphism*.

The following homomorphism, $f_{\text{epi}}:\mathscr{T}\to\mathscr{S}$, defines an *epimorphism*:



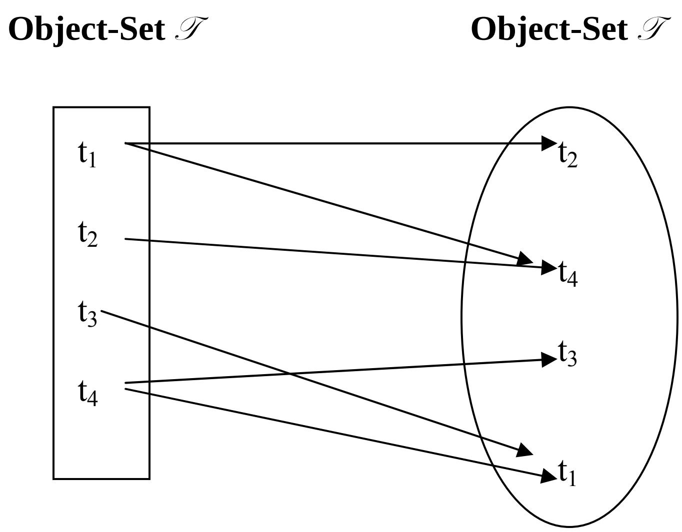
Isomorphism: A homomorphism and its inverse that are *bijective functions*; that is, functions that are both *one-to-one* and *onto*, is an *isomorphism*.

The following homomorphism, $f_{\text{iso}}:\mathscr{T} \rightarrow \mathscr{S}$, defines an *isomorphism*:



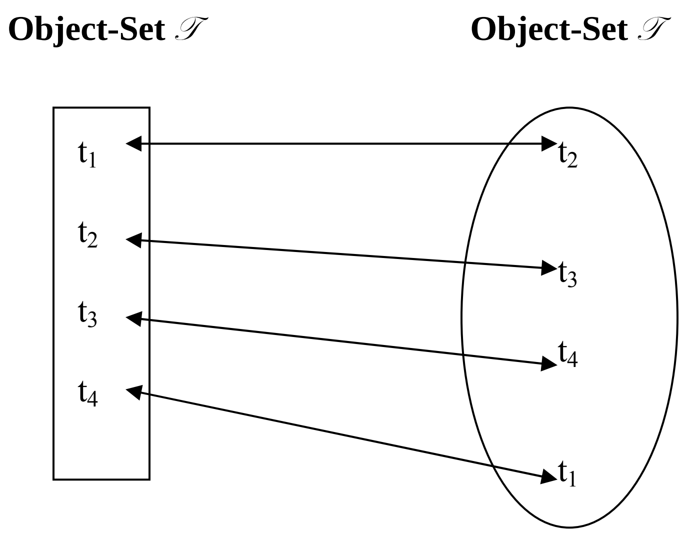
Endomorphism: A homomorphism that has a domain the same as its codomain, f:A→A, is an *endomorphism*.

The following homomorphism, $f_{\text{endo}}:\mathscr{T} \rightarrow \mathscr{T}$, defines an *endomorphism*:



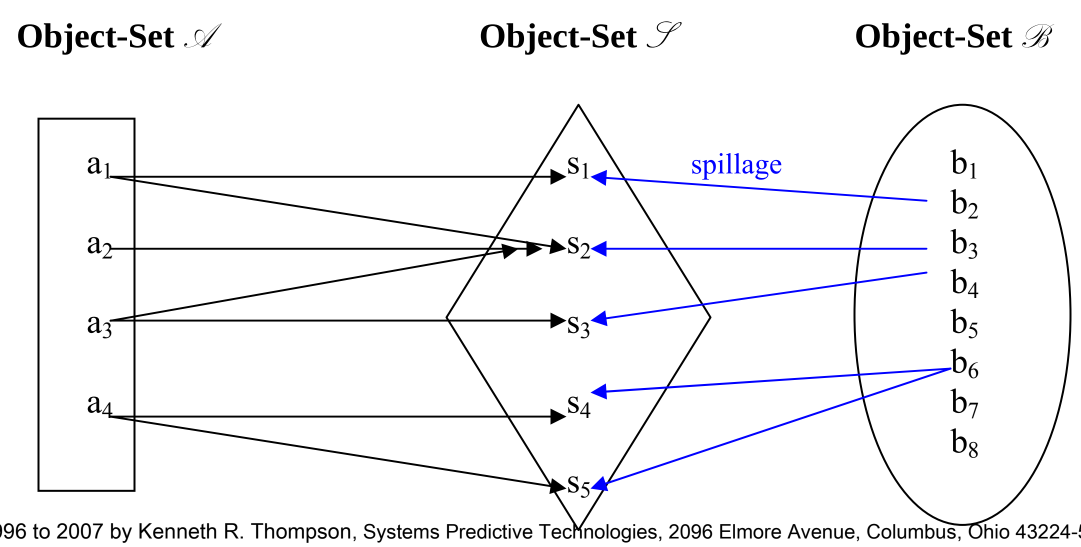
Automorphism: A homomorphism that is both an endomorphism and an isomorphism is an *automorphism*.

The following homomorphism, $f_{auto}:\mathscr{T}\rightarrow \mathscr{T}$, defines an *automorphism*:



Commensalmorphism: A homomorphism that is an epimorphism between the object-set of one system and the negasystem spillage of a coterminous system.

The following homomorphism, $f_{comm}:\mathscr{A}\rightarrow (\xi(\mathscr{B}))$, defines a *commensalmorphism*; where $\xi(\mathscr{B}) = \mathscr{S}$; that is, $\xi_{spillage}: \mathscr{B} \rightarrow \mathscr{S}$.



Symbiomorphism: A homomorphism and its inverse between coterminous systems.

The following homomorphisms, f and g, and the symbiotic-quantifier, $\mathsf{A}^{sym}\xi_{sym}$, define a *symbiomorphism*: $\mathsf{A}^{sym}\xi_{sym}(f:\mathscr{T}\rightarrow \mathscr{S}, g:\mathscr{S}\rightarrow \mathscr{T})\rightarrow(\mathscr{T}\cup\mathscr{S})$; where $\mathsf{A}^{sym}\xi_{sym}$ is true if the values of f and g map into $\mathscr{T}\cup\mathscr{S}$.

Object-Set $\mathscr{T}$ **Object-Set $\mathscr{S}$**